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Attorney's Docket No. 042933/298965

PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re: Nativadade Lobo Confirmation No.: 5615
Appl. No.: 09/625,201
Filed: July 21, 2000
For: PULSE SHAPING WHICH COMPENSATES FOR COMPONENT DISTORTION

BOX ISSUE FEE
Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

SUBMITTAL OF PRIORITY DOCUMENT

To complete the requirements of 35 U.S.C. § 119, enclosed is a certified copy of Great Britain priority Application No. 9801305.5, filed January 21, 1998.

Respectfully submitted,

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Tamara Stevens

We show that it is possible to design pulses to be used by a modulator that incorporates the current separation structure in such a way that the BER, spectral efficiency, amplitude envelope variation are traded off in a controlled way by ~~weighting~~ (i) developing separate cost functions for each (ii) weighting these ~~pulses~~ costs according to ~~import~~ design considerations. This is particularly desirable in the manufacture of mobile phones and designing ~~communication~~ systems

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ativity and innovation

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Newport
South Wales
NP10 8QQ

I, the undersigned, being an officer duly authorised in accordance with Section 74(1) and (4) of the Deregulation & Contracting Out Act 1994, to sign and issue certificates on behalf of the Comptroller-General, hereby certify that annexed hereto is a true copy of the documents as originally filed in connection with patent application GB9801305.5 filed on 21 January 1998.

In accordance with the Patents (Companies Re-registration) Rules 1982, if a company named in this certificate and any accompanying documents has re-registered under the Companies Act 1980 with the same name as that with which it was registered immediately before re-registration save for the substitution as, or inclusion as, the last part of the name of the words "public limited company" or their equivalents in Welsh, references to the name of the company in this certificate and any accompanying documents shall be treated as references to the name with which it is so re-registered.

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Signed

Dated 21 August 2007

Patents Form 1/77

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The Patent Office

G 1/77
23JAN98 E332519-1 002716
P01/7700 25.00 - 9801305.5

THE PATENT OFFICE
G
21 JAN 1998
LONDON

Request for grant of a patent

(See the notes on the back of this form. You can also get an explanatory leaflet from the Patent Office to help you fill in this form)

The Patent Office

Cardiff Road
Newport
Gwent NP9 1RH

1. Your reference — PAT 98003 GB

2. Patent application numb[illegible] *(The Patent Office will fill in t[illegible])* — **9801305.5** — **21 JAN 1998**

3. Full name, address and postcode of the or of each applicant *(underline all surnames)*

NOKIA MOBILE PHONES LIMITED
KEILALAHDENTIE 4
02150 ESPOO
FINLAND

Patents ADP number *(if you know it)*

If the applicant is a corporate body, give the country/state of its incorporation — FINLAND 5911995004

4. Title of the invention — OPTIMIZER

5. Name of your agent *(if you have one)*

"Address for service" in the United Kingdom to which all correspondence should be sent *(including the postcode)*

MRS HELEN LOUISE HAWS
NOKIA MOBILE PHONES
PATENT DEPARTMENT
ST GEORGES COURT
ST GEORGES ROAD
CAMBERLEY
SURREY GU15 3QZ UK

Patents ADP number *(if you know it)* 4105177001

6. If you are declaring priority from one or more earlier patent applications, give the country and the date of filing of the or of each of these earlier applications and *(if you know it)* the or each application number

Country	Priority application number *(if you know it)*	Date of filing *(day / month / year)*

7. If this application is divided or otherwise derived from an earlier UK application, give the number and the filing date of the earlier application

Number of earlier application	Date of filing *(day / month / year)*

8. Is a statement of inventorship and of right to grant of a patent required in support of this request? *(Answer 'Yes' if:*

a) any applicant named in part 3 is not an inventor, or
b) there is an inventor who is not named as an applicant, or
c) any named applicant is a corporate body.
See note (d))

YES

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G D Court.
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Patents Form 1/77

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Description	17
Claim(s)	
Abstract	
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10. If you are also filing any of the following, state how many against each item.

Priority documents	
Translations of priority documents	
Statement of inventorship and right to grant of a patent *(Patents Form 7/77)*	
Request for preliminary examination and search *(Patents Form 9/77)*	
Request for substantive examination *(Patents Form 10/77)*	
Any other documents *(please specify)*	

11. I/We request the grant of a patent on the basis of this application.

Signature H L HAWS Date 21 JANUARY 1998

H L HAWS AGENT FOR THE APPLICANT

12. Name and daytime telephone number of person to contact in the United Kingdom

HELEN HAWS 01276 419346

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Notes

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f) For details of the fee and ways to pay please contact the Patent Office.

```
Needs["LaurentFunctions`"]
```

```
RuleDelayed::rhs : Pattern t_ appears on the right-hand side of rule
    PhaseAngle[L_][t_] :→ (PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, <<1>>]).
```

```
Needs["LaurentNotationTest`"]
```

```
Needs::nocont : Context LaurentNotationTest` was not created when Needs was evaluated.
```

Information on the functions used can be obtained using help.

```
Names["LaurentFunctions`*"]
```

```
{AKN, AlphaKI, ANKInitialStateSetUp, BT, FiltPulse, h, hFiltered, InitialState, J,
 LaurentC, LaurentLK, LaurentS, M, ModulatingPulse, ModulationIndex, Modulator,
 NumberOfCurves, PhaseAngle, Receiver, ReceiverProper, S, SamplingInterval,
 StartingQuadrant, SyncSample, T, C, Φ, ψ}
```

$$\mathbf{T} := \frac{3}{812500}$$

```
BT := 0.3
```

$$\mathbf{ModulationIndex} := \frac{1}{2}$$

```
<< ModulatorData.m;
```

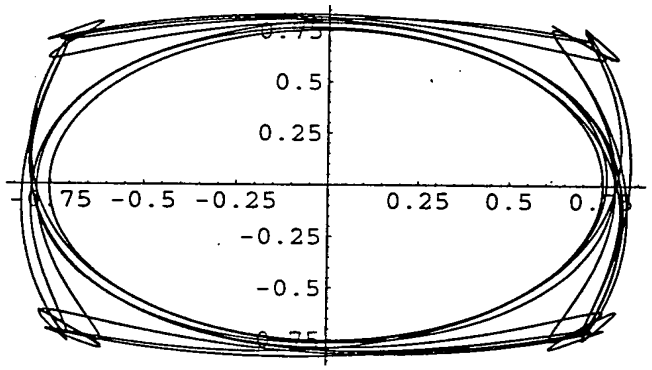
```
RandomBitSeq
```

```
{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
 -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1, 1, 1, -1,
 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1,
 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}
```

$$S_{NT+\Delta T} = \sum_{K=0}^{M-1} \sum_{n'=0}^{L_K-1} J^{A_{K,N-n'}} C_{K,n'T+\Delta T}$$

```
tom2 = Modulator[L][RandomBitSeq, NumberOfCurves → 2, ModulatingPulse → FiltPulse];
```

```
ListPlot[{Re[tom2], Im[tom2]} // Transpose, PlotJoined -> True]
```



- Graphics -

```
RandomBitSeq
```

```
{1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1,
 -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1, 1, 1, 1, -1,
 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1,
 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, 1, -1, 1, -1}
```

```
Receiver[L][tom2, StartingQuadrant -> 4]
```

```
{1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, -1, -1, 1, 1, 1, -1, -1,
 -1, 1, 1, 1, -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, -1, 1,
 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1,
 -1, -1, 1, -1, 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1}
```

We write all the possible terms

```
J^(λ0,N-5) ( J^(α_N-4 + α_N-3 + α_N-2 + α_N-1 + α_N) Pulse[0][δT] +
   J^(α_N-4 + α_N-3 + α_N-2 + α_N-1) Pulse[0][T + δT] + J^(α_N-4 + α_N-3 + α_N-2) Pulse[0][2 T + δT] +
   J^(α_N-4 + α_N-3) Pulse[0][3 T + δT] + J^α_N-4 Pulse[0][4 T + δT] + Pulse[0][5 T + δT] +
   J^(-α_N-5) Pulse[0][6 T + δT] + J^(-α_N-5 - α_N-6) Pulse[0][7 T + δT] +
   J^(-α_N-5 - α_N-6 - α_N-7) Pulse[0][8 T + δT] + J^(α_N-4 + α_N-3 + α_N-2 + α_N-1 + α_N - α_N-1) Pulse[1][δT] +
   J^(α_N-4 + α_N-3 + α_N-2 + α_N-1 - α_N-2) Pulse[1][T + δT] + J^(α_N-4 + α_N-3 + α_N-2 + α_N-3) Pulse[1][2 T + δT] +
   J^(α_N-4 + α_N-3 - α_N-4) Pulse[1][3 T + δT] + J^(α_N-4 - α_N-5) Pulse[1][4 T + δT] +
   J^(-α_N-6) Pulse[1][5 T + δT] + J^(-α_N-5 + α_N-7) Pulse[1][6 T + δT] )
```

```
( J^(α_N-4 + α_N-3 + α_N-2 + α_N-1 + α_N) Pulse[0][δT] +
   J^(α_N-4 + α_N-3 + α_N-2 + α_N-1) Pulse[0][T + δT] + J^(α_N-4 + α_N-3 + α_N-2) Pulse[0][2 T + δT] +
   J^(α_N-4 + α_N-3) Pulse[0][3 T + δT] + J^α_N-4 Pulse[0][4 T + δT] + Pulse[0][5 T + δT] +
   J^(-α_N-5) Pulse[0][6 T + δT] + J^(-α_N-5 - α_N-6) Pulse[0][7 T + δT] +
   J^(-α_N-5 - α_N-6 - α_N-7) Pulse[0][8 T + δT] + J^(α_N-4 + α_N-3 + α_N-2 + α_N-1 + α_N - α_N-1) Pulse[1][δT] +
   J^(α_N-4 + α_N-3 + α_N-2 + α_N-1 - α_N-2) Pulse[1][T + δT] +
   J^(α_N-4 + α_N-3 + α_N-2 + α_N-3) Pulse[1][2 T + δT] +
   J^(α_N-4 + α_N-3 - α_N-4) Pulse[1][3 T + δT] + J^(α_N-4 - α_N-5) Pulse[1][4 T + δT] +
   J^(-α_N-6) Pulse[1][5 T + δT] + J^(-α_N-5 + α_N-7) Pulse[1][6 T + δT] ) /. {α_N → x0,
  α_N-1 → x1, α_N-2 → x2, α_N-3 → x3, α_N-4 → x4, α_N-5 → x5, α_N-6 → x6, α_N-7 → x7}
```

```
J^(x0+x1+x2+x3+x4) Pulse[0][δT] + J^(x1+x2+x3+x4) Pulse[0][T + δT] +
 J^(x2+x3+x4) Pulse[0][2 T + δT] + J^(x3+x4) Pulse[0][3 T + δT] + J^x4 Pulse[0][4 T + δT] +
 Pulse[0][5 T + δT] + J^(-x5) Pulse[0][6 T + δT] + J^(-x5-x6) Pulse[0][7 T + δT] +
 J^(-x5-x6-x7) Pulse[0][8 T + δT] + J^(x0+x2+x3+x4) Pulse[1][δT] + J^(x1+x3+x4) Pulse[1][T + δT] +
 J^(x2+2 x3+x4) Pulse[1][2 T + δT] + J^x3 Pulse[1][3 T + δT] + J^(x4-x5) Pulse[1][4 T + δT] +
 J^(-x6) Pulse[1][5 T + δT] + J^(-x5+x7) Pulse[1][6 T + δT]
```

```
Clear[AbsoluteValue]
```

```
AbsoluteValue[Pulse_][{x0_, x1_, x2_, x3_, x4_, x5_, x6_, x7_}][δT_] :=
    Module[{a1, a2, a3, a4},
  a1 = J^(x0+x1+x2+x3+x4) Pulse[0][δT] +
     J^(x1+x2+x3+x4) Pulse[0][T + δT] + J^(x2+x3+x4) Pulse[0][2 T + δT] + J^(x3+x4) Pulse[0][3 T + δT] +
     J^x4 Pulse[0][4 T + δT] + Pulse[0][5 T + δT] + J^(-x5) Pulse[0][6 T + δT] +
     J^(-x5-x6) Pulse[0][7 T + δT] + J^(-x5-x6-x7) Pulse[0][8 T + δT] + J^(x0+x2+x3+x4) Pulse[1][δT] +
     J^(x1+x3+x4) Pulse[1][T + δT] + J^(x2+2 x3+x4) Pulse[1][2 T + δT] + J^x3 Pulse[1][3 T + δT] +
     J^(x4-x5) Pulse[1][4 T + δT] + J^(-x6) Pulse[1][5 T + δT] + J^(-x5+x7) Pulse[1][6 T + δT];
a2 =   ( Re[a1]^2 + Im[a1]^2 // ComplexExpand) /. {Im[x_] :> 0, Re[x_] :> x};
 a2 /. x_?NumberQ y_ -> Round[x] y]
```

```
AbsoluteValueInterfearingregions[Pulse_][
   {x0_, x1_, x2_, x3_, x4_, x5_, x6_, x7_}][δT_] :=
    Module[{a1, a2, a3, a4},
  a1 = J^(x0+x1+x2+x3+x4) Pulse[0][δT] + J^(x2+x3+x4) Pulse[0][2 T + δT] + J^(-x5) Pulse[0][6 T + δT] +
     J^(-x5-x6-x7) Pulse[0][8 T + δT] + J^(x1+x3+x4) Pulse[1][T + δT] + J^(-x6) Pulse[1][5 T + δT];
a2 =   ( Re[a1]^2 + Im[a1]^2 // ComplexExpand) /. {Im[x_] :> 0, Re[x_] :> x};
 a2 /. x_?NumberQ y_ -> Round[x] y]
```

```
AbsoluteValue[Pulse[8]][{-1, -1, -1, -1, -1, -1, -1, -1}][0.5 T]
```

$$x0[4]^2 + x0[12]^2 - 2\,x0[4]\,x0[20] + x0[20]^2 - 2\,x0[12]\,x0[28] + x0[28]^2 + 2\,x0[4]\,x0[36] - 2\,x0[20]\,x0[36] + x0[36]^2 + 2\,x0[12]\,x0[44] - 2\,x0[28]\,x0[44] + x0[44]^2 - 2\,x0[4]\,x0[52] + 2\,x0[20]\,x0[52] - 2\,x0[36]\,x0[52] + x0[52]^2 - 2\,x0[12]\,x0[60] + 2\,x0[28]\,x0[60] - 2\,x0[44]\,x0[60] + x0[60]^2 + 2\,x0[4]\,x0[68] - 2\,x0[20]\,x0[68] + 2\,x0[36]\,x0[68] - 2\,x0[52]\,x0[68] + x0[68]^2 + 2\,x0[12]\,x1[4] - 2\,x0[28]\,x1[4] + 2\,x0[44]\,x1[4] - 2\,x0[60]\,x1[4] + x1[4]^2 - 2\,x0[4]\,x1[12] + 2\,x0[20]\,x1[12] - 2\,x0[36]\,x1[12] + 2\,x0[52]\,x1[12] - 2\,x0[68]\,x1[12] + x1[12]^2 + 2\,x0[12]\,x1[20] - 2\,x0[28]\,x1[20] + 2\,x0[44]\,x1[20] - 2\,x0[60]\,x1[20] + 2\,x1[4]\,x1[20] + x1[20]^2 + 2\,x0[4]\,x1[28] - 2\,x0[20]\,x1[28] + 2\,x0[36]\,x1[28] - 2\,x0[52]\,x1[28] + 2\,x0[68]\,x1[28] - 2\,x1[12]\,x1[28] + x1[28]^2 + 2\,x0[12]\,x1[36] - 2\,x0[28]\,x1[36] + 2\,x0[44]\,x1[36] - 2\,x0[60]\,x1[36] + 2\,x1[4]\,x1[36] + 2\,x1[20]\,x1[36] + x1[36]^2 - 2\,x0[4]\,x1[44] + 2\,x0[20]\,x1[44] - 2\,x0[36]\,x1[44] + 2\,x0[52]\,x1[44] - 2\,x0[68]\,x1[44] + 2\,x1[12]\,x1[44] - 2\,x1[28]\,x1[44] + x1[44]^2 + 2\,x0[12]\,x1[52] - 2\,x0[28]\,x1[52] + 2\,x0[44]\,x1[52] - 2\,x0[60]\,x1[52] + 2\,x1[4]\,x1[52] + 2\,x1[20]\,x1[52] + 2\,x1[36]\,x1[52] + x1[52]^2$$

We set T = 1 to avoid precision problems.

```
T := 1

PulseSampling := T / 8

var = Join[Table[x0[i], {i, 0, LaurentLK[L][0] T / PulseSampling - 1}],
  Table[x1[i], {i, 0, LaurentLK[L][1] T / PulseSampling - 1}]];
```

```
initsol = Join[Table[LaurentC[8][0][i PulseSampling],
   {i, 0, LaurentLK[L][0] T / PulseSampling - 1}], Table[
   LaurentC[8][1][i PulseSampling], {i, 0, LaurentLK[L][1] T / PulseSampling - 1}]];
```

```
$Aborted
```

```
hessian = IdentityMatrix[Length[var]];
```

```
Pulse[8][0][y_] := x0[Round[y / PulseSampling] ];
Pulse[8][1][y_] := x1[Round[y / PulseSampling] ] ;
```

```
TempFun[x_][y_] := ((AbsoluteValue[Pulse[8]][x][y] - 1) ^ 2) // Expand;
```

```
TempFun2[x_][y_] := (AbsoluteValueInterfearingregions[Pulse[8]][x][y]);
```

```
TempFun[{-1, -1, -1, -1, -1, -1, -1, -1}][0.5 T];
```

```
ConvertToBitSeq[Depth_][Num_] :=
  Module[{bit, n},
  n = Num;
  ConvertNext := (bit = Mod[n, 2]; n = Floor[n / 2]; bit);
 ( Table[ConvertNext, {i, 1, Depth}] // Reverse) /. {0 → -1}];
```

```
PossSeq = Table[i, {i, 1, 2 ^ 8}] // Map[ConvertToBitSeq[8], #]&;
```

We calculate the

```
CostGenerator[TempFun_] :=
 Module[{Cost},
  Cost = 0;
PulseValue = 0;
CostUpdate := (Cost =
     Cost + Map[TempFun[#][PulseValue]&, PossSeq] // Apply[Plus, #]& // # / 256 &;
    PulseValue = PulseValue + PulseSampling);
Table[CostUpdate, {i, 1, T / PulseSampling // Round}]; Cost];
```

```
AmplitudeCost = CostGenerator[TempFun] * PulseSampling;
```

```
BERCost = CostGenerator[TempFun2] * PulseSampling;
```

```
Save["AmplitudeCost.m", {BERCost, AmplitudeCost, PulseSampling, T}]
```

In this section we find the bandwidth of the signal
Derivative of the Sync function is represented by

```
Clear[T, PulseSampling]
```

```
D[Sin[x]/x, x]
```

$$\frac{\text{Cos}[x]}{x} - \frac{\text{Sin}[x]}{x^2}$$

```
DSinc[0.0] := 0

DSinc[x_] /; x != 0 := Cos[x]/x - Sin[x]/x^2

<< AmplitudeCost.m;
```

The bandwidth is given by

```
TempInt0[m_, n_] :=
  (π/PulseSampling)^2 NIntegrate[DSinc[π/PulseSampling (x - m PulseSampling)]
    DSinc[π/PulseSampling (x - n PulseSampling)], {x, 0, 9 T}]
TempInt1[m_, n_] :=
  (π/PulseSampling)^2 NIntegrate[DSinc[π/PulseSampling (x - m PulseSampling)]
    DSinc[π/PulseSampling (x - n PulseSampling)],
   {x, 0, 7 T}]

Table[TempInt0[0, n], {n, 0, 5}]
```

```
General::stop :
 Further output of NIntegrate::ncvb will be suppressed during this calculation.
```

$$\{1.27879 \times 10^7, 5.06325, 10.6271, 9.78983, 4.51144, -2.28746\}$$

```
PulseSampling
```

$$\frac{1}{8}$$

```
ApproxBandWidth = Sum[(x0[i + 1] - x0[i])^2 / PulseSampling, {i, 0, 70}] +
  Sum[(x1[i + 1] - x1[i])^2 / PulseSampling, {i, 0, 54}];

BandWidth = Sum[x0[i] x0[j] TempInt0[i, j], {i, 0, 71}, {j, 0, 71}] +
  Sum[x1[i] x1[j] TempInt1[i, j], {i, 0, 55}, {j, 0, 55}];

Save["BandWidthCost.m", {BandWidth, T,}]

BandWidthCost = (ApproxBandWidth - 0.6)^2;
```

```
BERWeight = 0.3
BandWidthWeight = 0.4;
AmplitudeWeight = 1.0 - BandWidthWeight - BERWeight;
```

```
0.3
```

```
TotalCost = BandWidthWeight BandWidthCost + AmplitudeWeight AmplitudeCost +
  BERWeight * BERCost;
```

```
dir = Directory[];
SetDirectory["../new2satellite"];
<< DavidonFletcherPowell.m
SetDirectory[dir];
```

```
L := 8
```

```
DavidonFletcherPowell[TotalCost, var, hessian, initsol, 10];
```

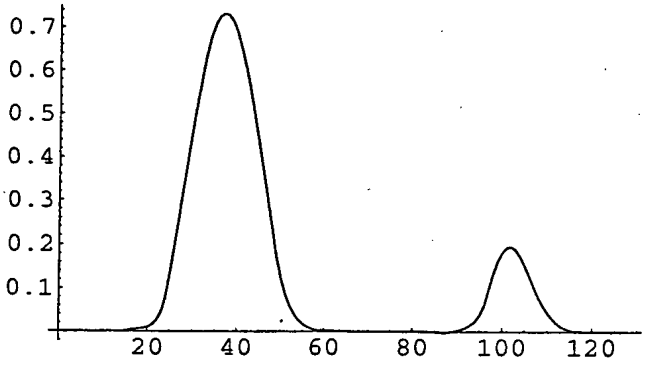
```
{0.0207507, {DavidonFletcherPowell`Private`alpha$39228→ 2.77153}}

{0.0164912, {DavidonFletcherPowell`Private`alpha$39228→ 2.45972}}

{0.0142842, {DavidonFletcherPowell`Private`alpha$39228→ 1.88521}}

{0.0126876, {DavidonFletcherPowell`Private`alpha$39228→ 2.08508}}

{0.0117343, {DavidonFletcherPowell`Private`alpha$39228→ 1.92086}}

{0.0112982, {DavidonFletcherPowell`Private`alpha$39228→ 1.55191}}

{0.0110916, {DavidonFletcherPowell`Private`alpha$39228→ 1.44356}}

{0.010974, {DavidonFletcherPowell`Private`alpha$39228→ 1.63222}}

{0.010886, {DavidonFletcherPowell`Private`alpha$39228→ 2.1399}}

{0.0108318, {DavidonFletcherPowell`Private`alpha$39228→ 2.14114}}
```

```
temp0[t_] = LaurentC[8][0][t];
```

```
temp1[t_] = LaurentC[8][1][t];
```

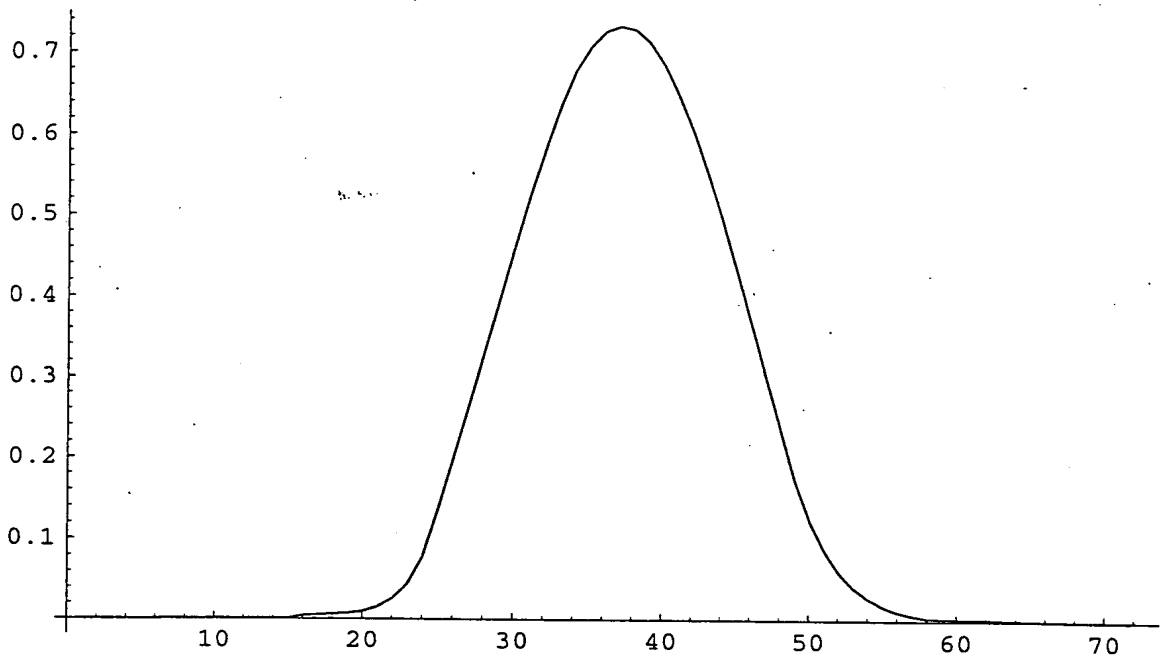
```
initsolC =
 Join[Table[temp0[i PulseSampling], {i, 0, LaurentLK[L][0] T / PulseSampling - 1}],
  Table[temp1[i PulseSampling], {i, 0, LaurentLK[L][1] T / PulseSampling - 1}]];
```

```
ListPlot[%278[[2]], PlotJoined -> True, PlotRange -> All]
```



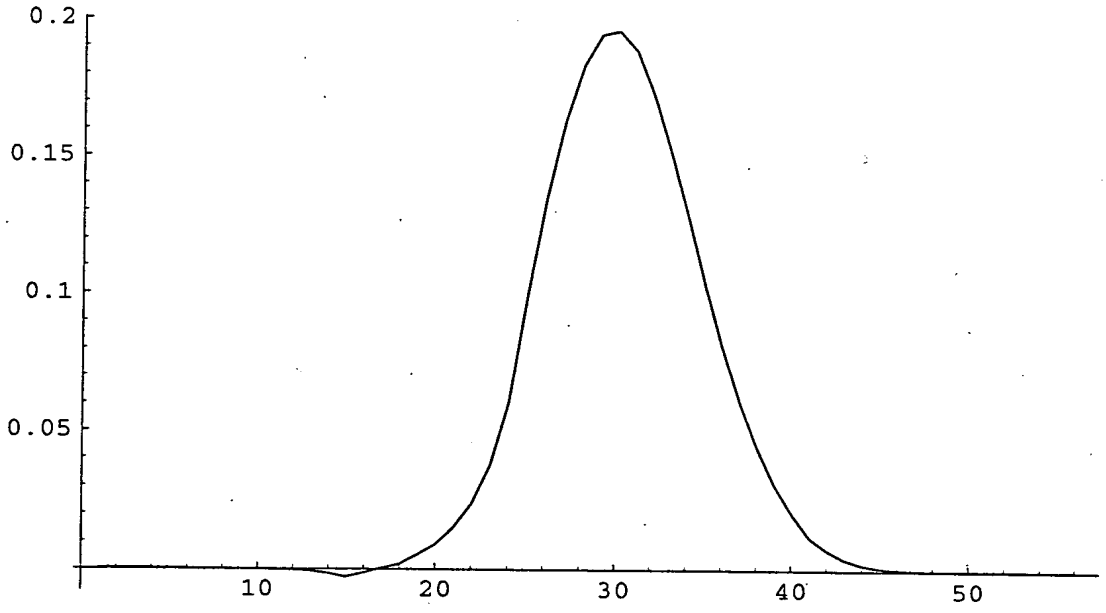
- Graphics -

```
ListPlot[%278[[2]] // Take[#, 72]&, PlotJoined -> True, PlotRange -> All]
```



- Graphics -

```
ListPlot[%278[[2]] // Take[#, -56]&, PlotJoined -> True, PlotRange -> All]
```



- Graphics -

```
{AmplitudeCost, BERCost, BandWidthCost, ApproxBandWidth} /. Thread[var -> initsol]
```

{0.0687008, 0.00375044, 0.0126982, 0.712686}

```
{AmplitudeCost, BERCost, BandWidthCost, ApproxBandWidth} /. Thread[var -> %278[[2]]]
```

{0.0208114, 0.00872422, 0.00492789, 0.670199}

```
(ApproxBandWidth - 0.6) ^ 2 /. Thread[var -> initsol]
```

```
0.0126982
```

```
(ApproxBandWidth - 0.6) ^ 2 /. Thread[var -> %278[[2]]]
```

```
0.00492789
```

```
OptPulse[8][0] = Interpolation[
  Table[{PulseSampling i, x0[i]}, {i, 0, LaurentLK[L][0] T / PulseSampling - 1}] /.
   Thread[var -> %278[[2]]]]
```

```
InterpolatingFunction[{{0, 8.875}}, <>]
```

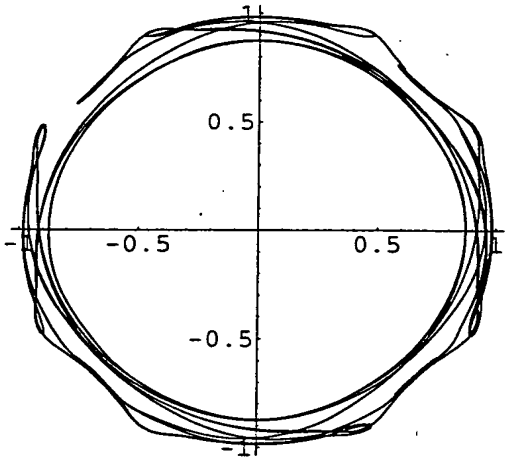
```
OptPulse[8][1] = Interpolation[
  Table[{PulseSampling i, x1[i]}, {i, 0, LaurentLK[L][1] T / PulseSampling - 1}] /.
   Thread[var -> %278[[2]]]]
```

```
InterpolatingFunction[{{0, 6.875}}, <>]
```

```
tom3 = Modulator[L][RandomBitSeq, NumberOfCurves → 2, ModulatingPulse → OptPulse];
```

```
InterpolatingFunction::dmval : Input value {221/32} lies outside the range of data
   in the interpolating function. Extrapolation will be used.

InterpolatingFunction::dmval : Input value {285/32} lies outside the range of data
   in the interpolating function. Extrapolation will be used.

InterpolatingFunction::dmval : Input value {111/16} lies outside the range of data
   in the interpolating function. Extrapolation will be used.

General::stop :
 Further output of InterpolatingFunction::dmval will be suppressed during this calculation.
```

```
ListPlot[{Re[tom3], Im[tom3]} // Transpose, PlotJoined -> True, AspectRatio -> 1]
```




```
- Graphics -
```

```
{AmplitudeCost, BERCost, BandWidthCost, ApproxBandWidth} /. Thread[var -> initsol]
```

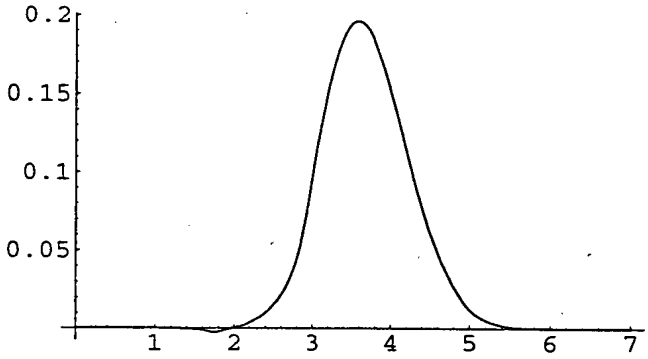
```
{0.0687008, 0.00375044, 0.0126982, 0.712686}
```

```
{AmplitudeCost, BERCost, BandWidthCost, ApproxBandWidth} /. Thread[var -> %278[[2]]]
```

```
{0.0208114, 0.00872422, 0.00492789, 0.670199}
```

```
Plot[%298[t], {t, 0, 7 T}, PlotRange -> All]
```

```
InterpolatingFunction::dmval : Input value {6.97422} lies outside the range of
   data in the interpolating function. Extrapolation will be used.

InterpolatingFunction::dmval : Input value {6.90039} lies outside the range of
   data in the interpolating function. Extrapolation will be used.

InterpolatingFunction::dmval : Input value {6.93624} lies outside the range of
   data in the interpolating function. Extrapolation will be used.

General::stop :
 Further output of InterpolatingFunction::dmval will be suppressed during this calculation.
```



- Graphics -

```
? Modulator
```

```
Modulator[L][BitSeq,Opts] assumes the following default options
  StartingQuadrant → 0,InitialState → Table[1,{i,1,20}],SamplingInterval → T/32,
  NumberOfCurves → 4,ModulatingPulse→ LaurentC . The Pulse is assumed to have
  the following structure Pulse[L][K][t]
```

```
? Interpolation
```

```
Interpolation[data] constructs an InterpolatingFunction object which represents an
  approximate function that interpolates the data. The data can have the forms
  {{x1, f1}, {x2, f2}, ... } or {f1, f2, ... }, where in the second case, the
  xi are taken to have values 1, 2, ... .
```

```
Table[

Thread[var -> %247[[2]]]

Clear[Pulse]

Sinc[x_] /; x == 0 := 1
Sinc[x_] /; x != 0 := Sin[x] / (x)

PulseSampling := T / 8

Pulse[8][0][y_] := x0[Round[y / PulseSampling]];
Pulse[8][1][y_] := x1[Round[y / PulseSampling]];

ModulationValue[FiltPulse[8]][{-1, -1, -1, -1, -1, -1, -1, -1}][0.5 T] // N
```

```
-0.754091 I
```

```
ConvertToBitSeq[Depth_][Num_] :=
  Module[{bit, n},
  n = Num;
  ConvertNext := (bit = Mod[n, 2]; n = Floor[n / 2]; bit);
 ( Table[ConvertNext, {i, 1, Depth}] // Reverse) /. {0 → -1}];
```

```
ConvertToBitSeq[8][2]
```

```
{-1, -1, -1, -1, -1, -1, 1, -1}
```

```
Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&;
```

```
Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&]
```

```
{{-1, -1, -1, -1, 1, -1, -1, -1},
 {-1, -1, -1, -1, 1, -1, -1, 1}, {-1, -1, -1, -1, 1, -1, 1, -1},
 {-1, -1, -1, -1, 1, -1, 1, 1}, {-1, -1, -1, -1, 1, 1, -1, -1},
 {-1, -1, -1, -1, 1, 1, -1, 1}, {-1, -1, -1, -1, 1, 1, 1, -1},
 {-1, -1, -1, -1, 1, 1, 1, 1}, {-1, -1, -1, 1, 1, -1, -1, -1},
 {-1, -1, -1, 1, 1, -1, -1, 1}, {-1, -1, -1, 1, 1, -1, 1, -1},
 {-1, -1, -1, 1, 1, -1, 1, 1}, {-1, -1, -1, 1, 1, 1, -1, -1}, {-1, -1, -1, 1, 1, 1, -1, 1},
 {-1, -1, -1, 1, 1, 1, 1, -1}, {-1, -1, -1, 1, 1, 1, 1, 1}, {-1, -1, 1, -1, 1, -1, -1, -1},
 {-1, -1, 1, -1, 1, -1, -1, 1}, {-1, -1, 1, -1, 1, -1, 1, -1},
 {-1, -1, 1, -1, 1, -1, 1, 1}, {-1, -1, 1, -1, 1, 1, -1, -1}, {-1, -1, 1, -1, 1, 1, -1, 1},
 {-1, -1, 1, -1, 1, 1, 1, -1}, {-1, -1, 1, -1, 1, 1, 1, 1}, {-1, -1, 1, 1, 1, -1, -1, -1},
 {-1, -1, 1, 1, 1, -1, -1, 1}, {-1, -1, 1, 1, 1, -1, 1, -1}, {-1, -1, 1, 1, 1, -1, 1, 1},
 {-1, -1, 1, 1, 1, 1, -1, -1}, {-1, -1, 1, 1, 1, 1, -1, 1}, {-1, -1, 1, 1, 1, 1, 1, -1},
 {-1, -1, 1, 1, 1, 1, 1, 1}, {-1, 1, -1, -1, 1, -1, -1, -1}, {-1, 1, -1, -1, 1, -1, -1, 1},
 {-1, 1, -1, -1, 1, -1, 1, -1}, {-1, 1, -1, -1, 1, -1, 1, 1},
 {-1, 1, -1, -1, 1, 1, -1, -1}, {-1, 1, -1, -1, 1, 1, -1, 1}, {-1, 1, -1, -1, 1, 1, 1, -1},
 {-1, 1, -1, -1, 1, 1, 1, 1}, {-1, 1, -1, 1, 1, -1, -1, -1}, {-1, 1, -1, 1, 1, -1, -1, 1},
 {-1, 1, -1, 1, 1, -1, 1, -1}, {-1, 1, -1, 1, 1, -1, 1, 1}, {-1, 1, -1, 1, 1, 1, -1, -1},
 {-1, 1, -1, 1, 1, 1, -1, 1}, {-1, 1, -1, 1, 1, 1, 1, -1}, {-1, 1, -1, 1, 1, 1, 1, 1},
 {-1, 1, 1, -1, 1, -1, -1, -1}, {-1, 1, 1, -1, 1, -1, -1, 1}, {-1, 1, 1, -1, 1, -1, 1, -1},
 {-1, 1, 1, -1, 1, -1, 1, 1}, {-1, 1, 1, -1, 1, 1, -1, -1}, {-1, 1, 1, -1, 1, 1, -1, 1},
 {-1, 1, 1, -1, 1, 1, 1, -1}, {-1, 1, 1, -1, 1, 1, 1, 1}, {-1, 1, 1, 1, 1, -1, -1, -1},
 {-1, 1, 1, 1, 1, -1, -1, 1}, {-1, 1, 1, 1, 1, -1, 1, -1}, {-1, 1, 1, 1, 1, -1, 1, 1},
 {-1, 1, 1, 1, 1, 1, -1, -1}, {-1, 1, 1, 1, 1, 1, -1, 1}, {-1, 1, 1, 1, 1, 1, 1, -1},
 {-1, 1, 1, 1, 1, 1, 1, 1}, {1, -1, -1, -1, 1, -1, -1, -1}, {1, -1, -1, -1, 1, -1, -1, 1},
 {1, -1, -1, -1, 1, -1, 1, -1}, {1, -1, -1, -1, 1, -1, 1, 1},
 {1, -1, -1, -1, 1, 1, -1, -1}, {1, -1, -1, -1, 1, 1, -1, 1}, {1, -1, -1, -1, 1, 1, 1, -1},
 {1, -1, -1, -1, 1, 1, 1, 1}, {1, -1, -1, 1, 1, -1, -1, -1}, {1, -1, -1, 1, 1, -1, -1, 1},
 {1, -1, -1, 1, 1, -1, 1, -1}, {1, -1, -1, 1, 1, -1, 1, 1}, {1, -1, -1, 1, 1, 1, -1, -1},
 {1, -1, -1, 1, 1, 1, -1, 1}, {1, -1, -1, 1, 1, 1, 1, -1}, {1, -1, -1, 1, 1, 1, 1, 1},
 {1, -1, 1, -1, 1, -1, -1, -1}, {1, -1, 1, -1, 1, -1, -1, 1}, {1, -1, 1, -1, 1, -1, 1, -1},
 {1, -1, 1, -1, 1, -1, 1, 1}, {1, -1, 1, -1, 1, 1, -1, -1}, {1, -1, 1, -1, 1, 1, -1, 1},
 {1, -1, 1, -1, 1, 1, 1, -1}, {1, -1, 1, -1, 1, 1, 1, 1}, {1, -1, 1, 1, 1, -1, -1, -1},
 {1, -1, 1, 1, 1, -1, -1, 1}, {1, -1, 1, 1, 1, -1, 1, -1}, {1, -1, 1, 1, 1, -1, 1, 1},
 {1, -1, 1, 1, 1, 1, -1, -1}, {1, -1, 1, 1, 1, 1, -1, 1}, {1, -1, 1, 1, 1, 1, 1, -1},
 {1, -1, 1, 1, 1, 1, 1, 1}, {1, 1, -1, -1, 1, -1, -1, -1}, {1, 1, -1, -1, 1, -1, -1, 1},
 {1, 1, -1, -1, 1, -1, 1, -1}, {1, 1, -1, -1, 1, -1, 1, 1}, {1, 1, -1, -1, 1, 1, -1, -1},
 {1, 1, -1, -1, 1, 1, -1, 1}, {1, 1, -1, -1, 1, 1, 1, -1}, {1, 1, -1, -1, 1, 1, 1, 1},
 {1, 1, -1, 1, 1, -1, -1, -1}, {1, 1, -1, 1, 1, -1, -1, 1}, {1, 1, -1, 1, 1, -1, 1, -1},
 {1, 1, -1, 1, 1, -1, 1, 1}, {1, 1, -1, 1, 1, 1, -1, -1}, {1, 1, -1, 1, 1, 1, -1, 1},
 {1, 1, -1, 1, 1, 1, 1, -1}, {1, 1, -1, 1, 1, 1, 1, 1}, {1, 1, 1, -1, 1, -1, -1, -1},
 {1, 1, 1, -1, 1, -1, -1, 1}, {1, 1, 1, -1, 1, -1, 1, -1}, {1, 1, 1, -1, 1, -1, 1, 1},
 {1, 1, 1, -1, 1, 1, -1, -1}, {1, 1, 1, -1, 1, 1, -1, 1}, {1, 1, 1, -1, 1, 1, 1, -1},
 {1, 1, 1, -1, 1, 1, 1, 1}, {1, 1, 1, 1, 1, -1, -1, -1}, {1, 1, 1, 1, 1, -1, -1, 1},
 {1, 1, 1, 1, 1, -1, 1, -1}, {1, 1, 1, 1, 1, -1, 1, 1}, {1, 1, 1, 1, 1, 1, -1, -1},
 {1, 1, 1, 1, 1, 1, -1, 1}, {1, 1, 1, 1, 1, 1, 1, -1}, {1, 1, 1, 1, 1, 1, 1, 1}}
```

```
x1 = Map[ModulationValue[FiltPulse[8]][#][0.5 T]&,
    Select[Table[i, {i, 1, 2^8}] // Map[ConvertToBitSeq[8], #]&, (#[[5]] == 1)&] ]
   (-I)
```

```
{0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
 0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
 0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
 0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
 0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
 0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
 0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
 0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091,
 0.749266, 0.74922, 0.749173, 0.749219, 0.711482, 0.711529, 0.711482, 0.711435,
 0.829796, 0.82975, 0.829703, 0.829749, 0.792012, 0.792059, 0.792012, 0.791965,
 0.776885, 0.776839, 0.776791, 0.776838, 0.739101, 0.739147, 0.7391, 0.739054,
 0.802178, 0.802131, 0.802084, 0.80213, 0.764394, 0.76444, 0.764393, 0.764347,
 0.759521, 0.759475, 0.759428, 0.759474, 0.721738, 0.721784, 0.721737, 0.72169,
 0.819541, 0.819495, 0.819447, 0.819494, 0.781757, 0.781804, 0.781756, 0.78171,
 0.78714, 0.787094, 0.787047, 0.787093, 0.749356, 0.749403, 0.749355, 0.749309,
 0.791922, 0.791876, 0.791829, 0.791875, 0.754138, 0.754185, 0.754138, 0.754091}
```

```
Limit[D[Sin[x] / x, x], x -> 0]
```

```
0
```

```
? Limit
```

```
Limit[expr, x->x0] finds the limiting value of expr when x approaches x0.
```

$$\text{Map}\left[\left(1/2 - \sigma/2\,\text{Erf}\left[\frac{\#}{\sigma}\right]\right)\&,\ x1\right] // \text{Apply}[\text{Plus}, \#]\& // \#/128\&;$$

```
Ber[σ_] := Evaluate[%48]
```

```
Ber[0.01]
```

```
0.495
```

```
Erf[Infinity]
```

```
1
```

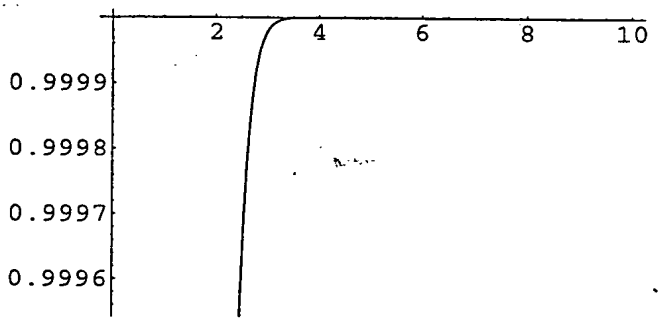
```
D[Erf[x], x]
```

$$\frac{2\,E^{-x^2}}{\sqrt{\pi}}$$

```
? Erf
```

```
Erf[z] gives the error function erf(z). Erf[z0, z1] gives the generalized error
  function erf(z1) - erf(z0).
```

```
Plot[Erf[x], {x, 0, 10}]
```



- Graphics -

```
? FiltPulse
```

Global`FiltPulse

```
T := 3 / 812500
```

Null

```
?? T
```

This is the symbol period

```
?? T
```

This is the symbol period

T := 3 / 812500

```
Clear[T, x0, x1, x2, x3, x4, x5, x6, x7]
```

```
BeginPackage["LaurentFunctions`"]

T::usage = "This is the symbol period"

BT::usage = "This is the usual product"

h::usage = "This is the raw gaussian pulse"

Φ::usage = "This denotes modulation index Pi"

ψ::usage = "ψ[L,t] is Laurents function"

hFiltered::usage = "This is the filtered gaussian pulse"

PhaseAngle::usage = "This function takes some time to calculate. The following
   code will draw a graph of the functionPhaseFunction[t_] =N[ ϕL,t];
phasepoints =Table[{t,PhaseFunction[t T]}//N,{t,0,L ,1/40}];
ListPlot[phasepoints,PlotJoined -> True]"

S ::usage = "S = Sin[Φ]"

J::usage = "J = e^(j Φ)"

C::usage = "C = Cos[Φ]"

M::usage = "M = 2^(L-1)"

ModulationIndex::usage = "ModulationIndex = h"

LaurentS::usage = "LaurentS[L][n][t] = Sin[ ψ[L,t + n T]]/ S"

LaurentC::usage = "LaurentC[L][K][t] is Laurents CK,t"

AlphaKI::usage = "AlphaKI[LL][K,i] is Laurents αK,i"

LaurentLK::usage = "LaurentLK[L][K] gives the support of CK,t"
```

The start of Modulator Definitions

```
ANKInitialStateSetUp::usage =
 "ANKInitialStateSetUp[L][K][InitBitSeq,AccumulatedPhase] sets up the sequence
   of prior states of AK,N that the modulator went thgough to get to the
   constelation point specified by  AccumulatedPhase which is really A0,0"

AKN::usage = "AKN[L][K][{State,AccumulatedPhase}] defines AK,N in terms of A0,N"

ModulatingPulse::usage =
 "The Pulse is assumed to have the following structure Pulse[L][K][t]"

NumberOfCurves::usage = "The number of pulses used by the modulator"

SamplingInterval::usage =
 "SamplingInterval is the interval between samples of the output of the modulator"

InitialState::usage = "InitialState is the set of bits that are assumed
   to be present before i.e {a-1,a-2, ...}"

StartingQuadrant::usage = "StartingQuadrant = A0,-1and is a number"

Modulator::usage = "Modulator[L][BitSeq,Opts] assumes the following
   default options StartingQuadrant → 0,InitialState → Table[1,{i,
   1,20}],SamplingInterval → T/32,NumberOfCurves → 4,ModulatingPulse→
   LaurentC . The Pulse is assumed to have the following structure Pulse[L][K][t]"
```

Start of the Receiver Functions

```
FiltPulse::usage = "The default pulse and is called by FiltPulse[L][K][t]"

SyncSample::usage = "Given that the sampling interval is T/32,then sync
   sample has rang -16 to 16 and this moves the point used to demodulate "

Receiver::usage = " Receiver[L][InputSeq,Opts] assumes the following
   default options {StartingQuadrant→0,InitialState→{1,1,1,1,1,1,1,1,
   1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1},SamplingInterval -> T/32,ModulatingPulse →
   FiltPulse,SyncSample -> 0,NumberOfCurves -> 2    "

ReceiverProper::usage = "ReceiverProper[L][ StartingQuadrant, InitialState,
   SamplingInterval,ModulatingPulse,SyncSample,NumberOfCurves,InputSeq]
   is called by Receiver after all the options have been resolved"

Begin["`Private`"]
```

$$\sigma := \frac{\sqrt{\text{Log}[2]}}{2\,\pi\,\text{BT}}$$

$$\text{h}[\text{t\_}] := \frac{\text{Exp}\left[-\frac{\text{t}^2}{2\,\sigma^2\,\text{T}^2}\right]}{\sqrt{2\,\pi}\;\sigma\;\text{T}}$$

Ideally we would and did define the effect of the convolution of h[t] by the formula below. However, this version of *Mathematica* gives an error.

$$\text{h}_{\text{fil}}[\text{t\_}] := \text{Release}\left[\text{Module}\left[\{\tau\},\ \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \frac{\text{h}[\text{t}-\tau]}{\text{T}}\,\text{d}\tau\right]\right]$$

```
hFiltered[PulseWidth_][t_] := Module[{x1, x2, x3}, x1 =
    (N[#1, 40]&)[Table[{t1, ∫_(-τ/2)^(τ/2) h[t1 - τ]/T dτ}, {t1, -PulseWidth/2, PulseWidth/2, T/20}]];
  x2 = Interpolation[x1]; x2[t]]

Φ := N[ModulationIndex π]

C := Cos[Φ];
S := Sin[Φ];
J := (e^(j Φ) // Chop);
M := 2^(L-1);

PhaseAngle[L_][t_] /; t ≤ 0 := 0

PhaseAngle[L_][t_] /; t ≥ L T := Φ

PhaseAngle[L_][t_] :=
 PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, x1 = hFiltered[3 L T][t1 - L T/2];
   x2 = Table[{t2, Φ ∫_(-L T)^t2 Evaluate[x1] dt1}, {t2, 0, L T, T/100}]; Interpolation[x2][t]]

ψ[L_, t_] /; 0 < t < L T := PhaseAngle[L][t]

ψ[L_, t_] /; 2 L T > t ≥ L T := Φ - PhaseAngle[L][t - L T]
```

We need to put this to avoid the function being extrapolated

```
ψ[L_ , t_] /; ! (0 < t < L T) && ! (2 L T > t ≥ L T) := 0

LaurentS[L_][n_][t_] := Sin[ψ[L, t + n T]] / S
```

```
AlphaKI[LL_][K_, i_] /; (0 < i < LL) && (0 <= K < 2^(LL-1)) :=
 Module[{x1, x2, x3, KNum}, x1 := (x2 = Mod[KNum, 2]; KNum = (KNum - x2)/2; x2);
  KNum = K; x3 = Table[x1, {ii, 0, LL - 1}]; x3[[i]]]

LaurentLK[L_][K_] :=
 Module[{x1}, x1 = Table[L (2 - AlphaKI[L][K, ii]) - ii, {ii, 1, L - 1}]; Min[x1]]

LaurentC[L_][K_][t_] /; 0 <= K < 2^L :=
 LaurentS[L][0][t] Product[LaurentS[L][ii + L AlphaKI[L][K, ii]][t], {ii, 1, L-1}]
```

The start of the modulator function

```
ANKInitialStateSetUp[L_][K_][InitBitSeq_, AccumulatedPhase_] :=
          Module[{x1, x2, x3, x4, x5, acuphase, initbitseq},
  initbitseq = InitBitSeq;
  acuphase = AccumulatedPhase;
   UpdateSeq :=
   Module[{},  x1 = acuphase - Sum[initbitseq[[i]] AlphaKI[L][K, i], {i, 1, L - 1}];
    acuphase =  acuphase - First[initbitseq]; initbitseq = Rest[ initbitseq];  x1];
  Table[ UpdateSeq, {i, 1, LaurentLK[L][K]}]]

AKN[L_][K_][{State_, AccumulatedPhase_}] :=
 AccumulatedPhase - Sum[State[[i + 1]] AlphaKI[L][K, i], {i, 1, L - 1}]

Options[Modulator] := {StartingQuadrant → 0, InitialState → Table[1, {i, 1, 20}],
  SamplingInterval → T / 32, NumberOfCurves → 4, ModulatingPulse → LaurentC}

Modulator[L_][BitSeq_, Opts___] :=
 Module[
  {x1, x2, x3, x4, x5, x6, state, AccumalatedPhase, seq, AKNState, Curves, Pulse},
x1 = SamplingInterval /. {Opts} /. Options[Modulator];
state = InitialState /. {Opts} /. Options[Modulator];
x3 = StartingQuadrant /. {Opts} /. Options[Modulator];
x4 = SamplingInterval /. {Opts} /. Options[Modulator];
Pulse = ModulatingPulse /. {Opts} /. Options[Modulator];
Curves = (NumberOfCurves /. {Opts} /. Options[Modulator] ) - 1;
AccumulatedPhase = x3 ;
seq = BitSeq;
Table[
   AKNState[K] = ANKInitialStateSetUp[L][K][state, AccumulatedPhase], {K, 0, Curves}];
x5 := Module[{}, state = Join[{First[seq]}, Drop[state, -1]];
AccumulatedPhase = AccumulatedPhase + First[seq];
seq = Rest[seq];
Table[AKNState[K] = Join[{AKN[L][K][{state, AccumulatedPhase}]},
      Drop[AKNState[K], -1]], {K, 0, Curves}];
x6[τ_] = Sum[Sum[ (J)^AKNState[K][[i + 1]] Pulse[L][K][τ + i T],
      {i, 0, LaurentLK[L][K] - 1}], {K, 0, Curves}];
Table[x6[τ], {τ, 0, T - x4, x4}]];
 Table[x5, {kk, 1, Length[BitSeq]}] // Flatten]

Options[Receiver] := {StartingQuadrant → 0,
  InitialState → {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, SamplingInter
  ModulatingPulse → FiltPulse, SyncSample -> 0, NumberOfCurves -> 2};

Receiver[L_][InputSeq_, Opts___] :=
  Module[{x1, x2, x3, x4, x5, x6},
  x1 = StartingQuadrant /. {Opts} /. Options[Receiver];
  x2 = InitialState /. {Opts} /. Options[Receiver];
  x3 = SamplingInterval /. {Opts} /. Options[Receiver];
  x4 = ModulatingPulse /. {Opts} /. Options[Receiver];
  x5 = SyncSample /. {Opts} /. Options[Receiver];
  x6 = NumberOfCurves /. {Opts} /. Options[Receiver];
ReceiverProper[L][x1, x2, x3, x4, x5, x6, InputSeq]]
```

```
ReceiverProper[L_][ StartingQuadrant_, InitialState_, SamplingInterval_,
  ModulatingPulse_, SyncSample_, NumberOfCurves_, InputSeq_] :=
     Module[{x1, sgn, ReceivedSeq,  ExpectedValue, seq, ReceiveNext, 𝔻, 𝔄},
  x1 = T / SamplingInterval;
  ReceivedSeq = Partition[InputSeq, x1] // Transpose // #[[SyncSample + 1]]&;
  ExpectedValue =
   ModulatingPulse[L][0][(LaurentLK[L][0] / 2) T + SyncSample SamplingInterval];
  𝔄 = StartingQuadrant;
  sgn = 0;
  𝔻 = {};
  seq = ReceivedSeq;
  ReceiveNext :=
  Module[{x1, x2},
    x1 = ( (-1)^sgn J^𝔄 First[seq]) // Im;
   If[Abs[x1 - ExpectedValue] <= Abs[x1 + ExpectedValue],
     𝔻 = Join[ 𝔻, {1}];  𝔄 = 𝔄 + 1,  𝔻 = Join[ 𝔻, {-1}];  𝔄 = 𝔄 - 1 ];
    seq = Rest[seq]; sgn = Mod[sgn + 1, 2]];
 Table[ReceiveNext, {i, 1, Length[ReceivedSeq]}];
  𝔻]

End[]

EndPackage[]
```